Calculation of Electric Dipole Moments of the Nucleon

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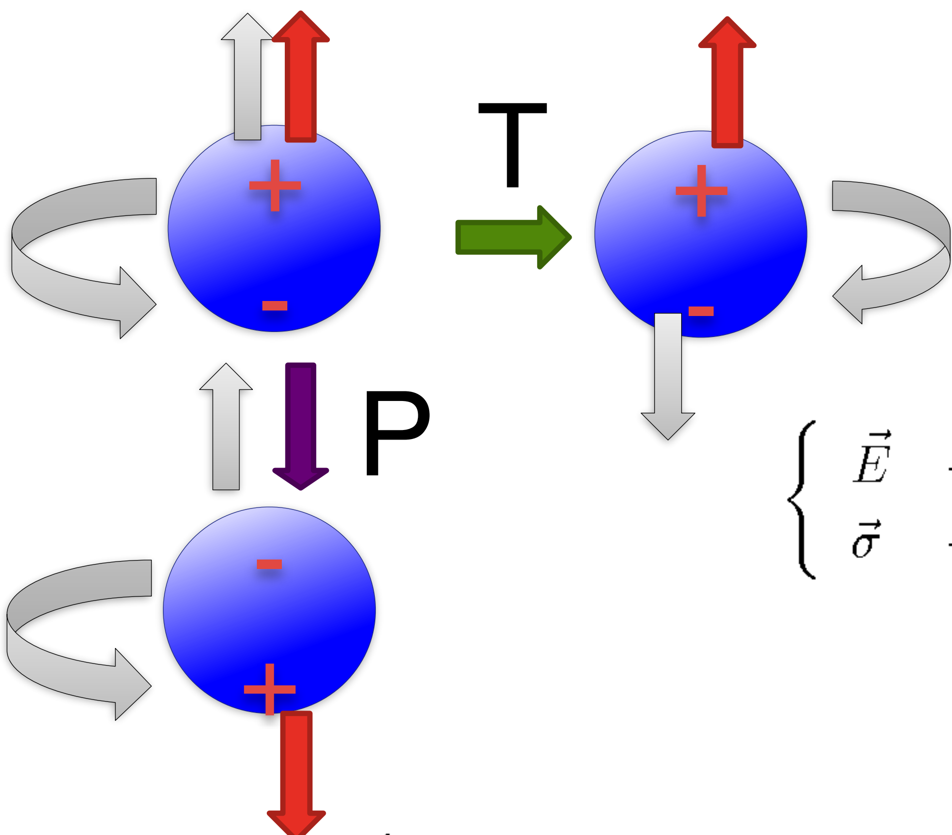
Introduction

- Electric Dipole Moment d
  Energy shift of a spin particle in an electric field
- Non-zero EDM : P&T (CP through CPT) violation



$$H_{\mathsf{EDM}} = -d\,\langle\vec{\sigma}\rangle\cdot\vec{E}$$

$$\begin{cases} \vec{E} \xrightarrow{\mathrm{T}} \vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{T}} -\vec{\sigma} \end{cases}$$

→ ***H***

$\rightarrow$ ***H_{EDM}* is CP-odd !**

$$\begin{cases} \vec{E} \xrightarrow{\mathrm{P}} -\vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{P}} \vec{\sigma} \end{cases}$$

$\rightarrow$ ***H_{EDM}* is P-odd**

Nucleon EDM Experiments

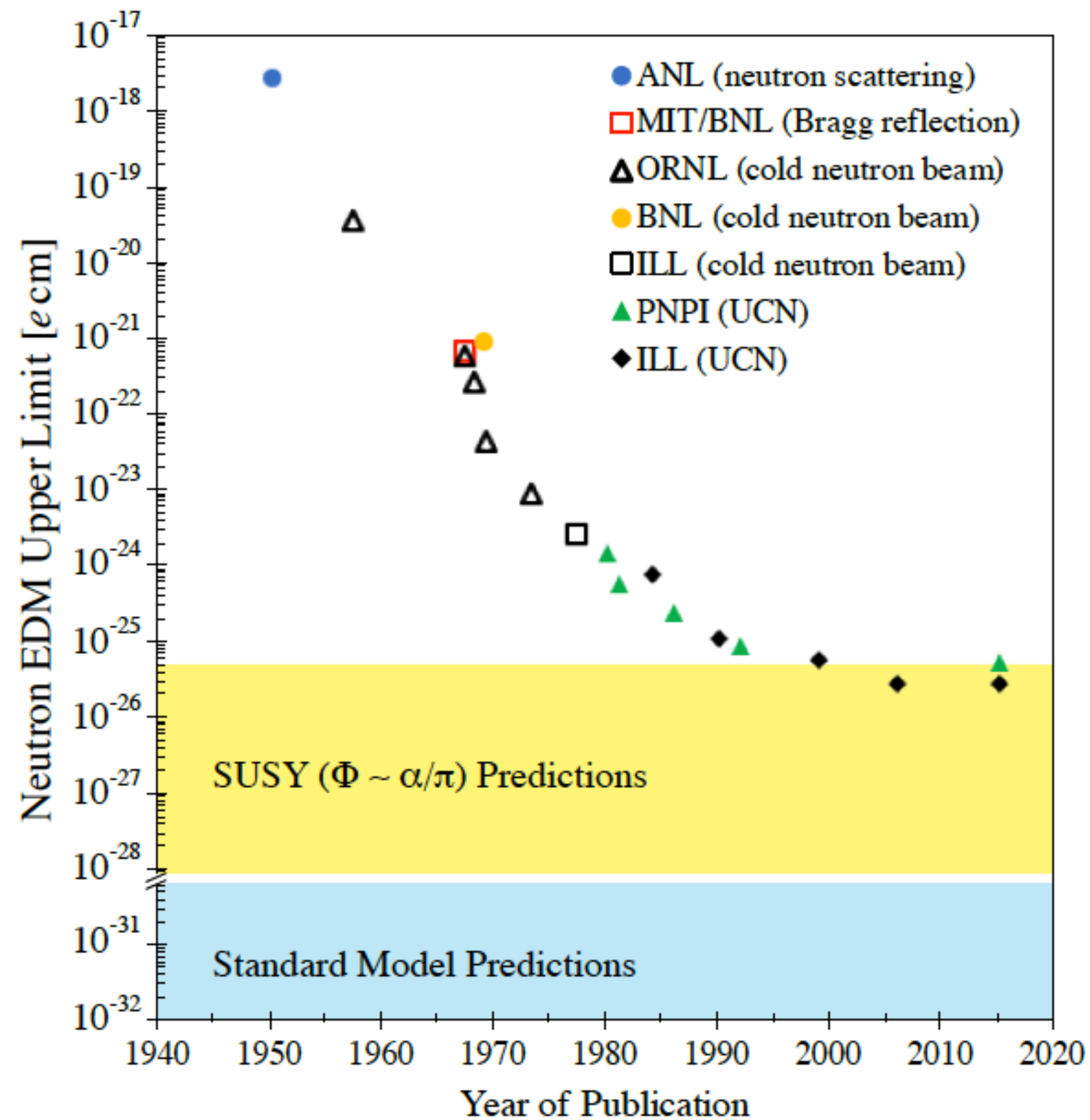
Current nEDM limits:

^{199}Hg spin precession (UW) [Graner et al, 2016]
Ultracold Neutrons in a trap (ILL) [Baker 2006]

$$|d_{Hg}| < 7.4 \times 10^{-30}\ \mathrm{e \cdot cm}$$

$$|d_n| < 2.6 \times 10^{-26}\ \mathrm{e \cdot cm}$$

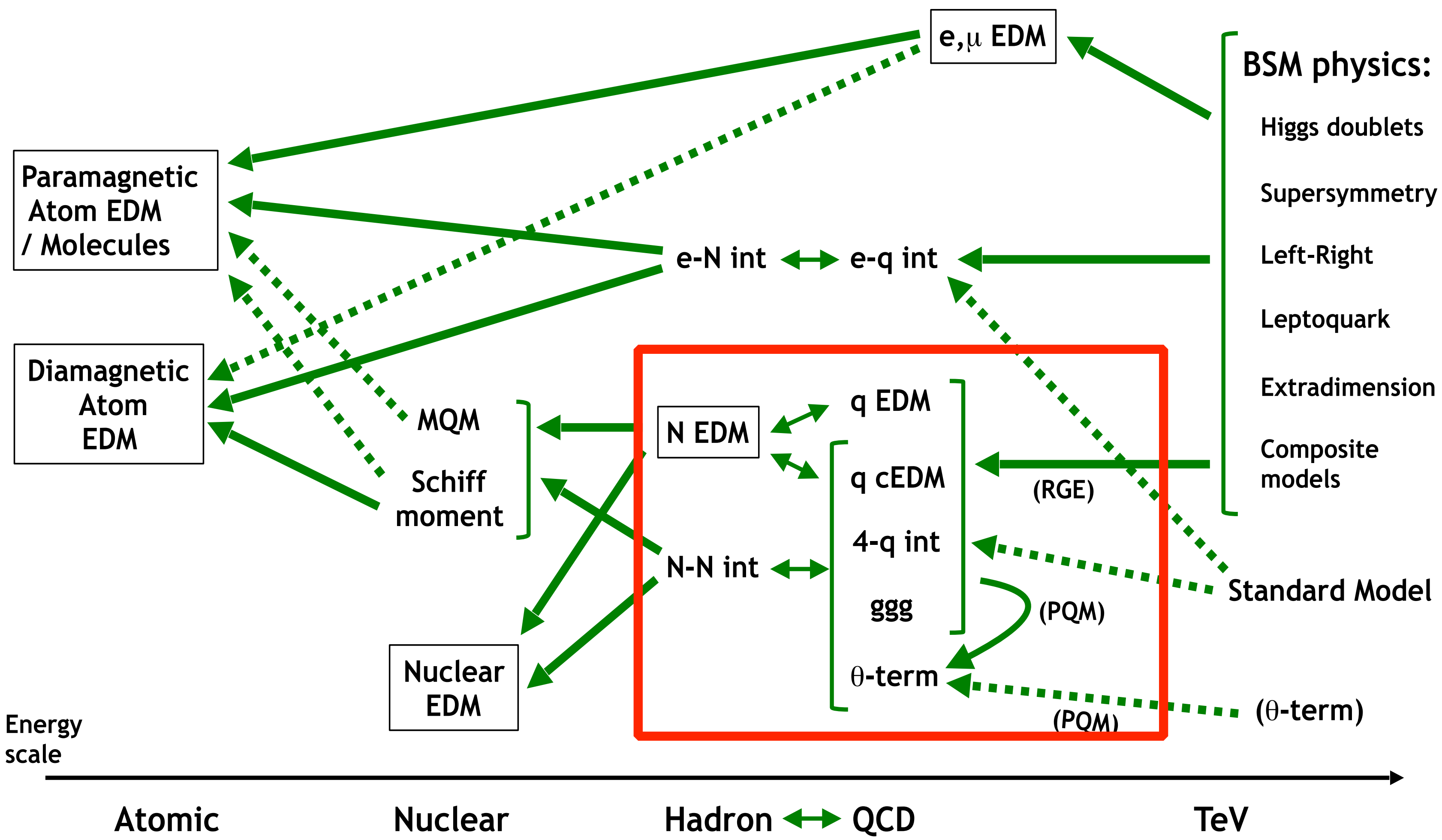
	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	*< 0.001*

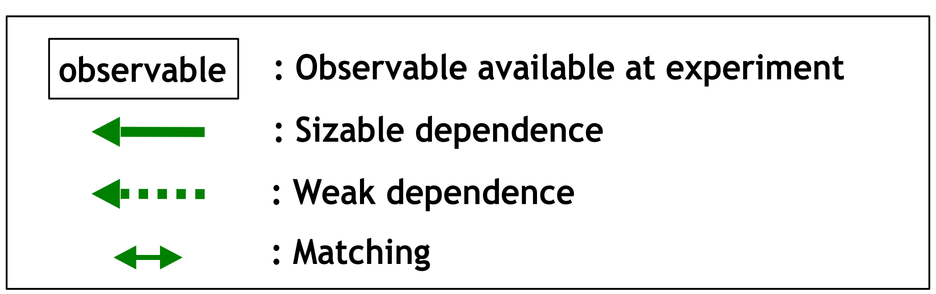


Figures from S.Kawasaki (KEK)

Nucleon EDM

[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]





Important bottleneck of the EDM calculation!

Role of (lattice) QCD : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions (Form factor, dn)

Non-perturbative determination is important

→ Lattice QCD calculation!

Effective CPV operators

$$\mathcal{L}_{eff}^{\cancel{CP}} = \frac{g_s^2}{32\pi^2}\bar{\theta}G_{\mu\nu}\tilde{G}^{\mu\nu}$$ dim=4, θ_{QCD}

$$-\frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\bar{\psi}_i G\cdot\sigma\gamma_5\psi_i$$ dim=5, chromo EDM

$$-\frac{i}{2}\sum_{i=e,u,d,s}d_i\bar{\psi}_i F\cdot\sigma\gamma_5\psi_i$$ dim=5, e, quark EDM

$$+\,\omega f^{abc}G_{\mu\nu,a}G^{\mu\beta,b}G^{\nu,c}_{\ \ \beta}$$ dim=6, Weinberg three gluon

$$+\sum C_i^{(4q)}\mathcal{O}_i^{(4q)}$$ dim=6, Four-quark operators

$\bar{\theta} \le \mathcal{O}(10^{-10})$ **: Strong CP problem**

quark-chromo EDM Dim=5 operators suppressed by m_q/Λ^2 **→ effectively dim=6,**

quark EDM … the most accurate lattice data for EDM (~5% for u,d)

cEDM and Weinberg ops. are ongoing. [T. Bhattacharya, plenary talk]

Lattice QCD calculations are important to constrain θ, cEDM etc.

Calculating CP-odd interaction on the lattice

CP-violating interaction on lattice

- Linearization of CP-odd interaction (e.g. : θ-EDM)

$$e^{-S_{QCD}-i\theta Q} = e^{-S_{QCD}}\left[1 - i\theta Q + \mathcal{O}(\theta^2)\right]$$

$$\langle \mathcal{O}\rangle_{\cancel{CP}} = \langle \mathcal{O}\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\rangle_{CP-even} + \mathcal{O}(\theta^2)$$

(CP-even) (CP-odd)

CPV operator : Q, cEDM, etc..., θ << 1

- P, T-odd form factor [E. Shintani et al 2005, F. Berruto et al 2015, A. Schindler et al, 2015, C. Alexandra et al, 2015, J. Dragos et al, 2019]

$$\langle p',\sigma'|J^\mu|p,\sigma\rangle = \bar{u}_{p',\sigma'}\left[\underbrace{F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}}_{\text{P, T even}} - \underbrace{F_3(Q^2)\frac{\gamma_5\sigma^{\mu\nu}q_\nu}{2m_N}}_{\text{P, T odd}}\right]u_{p,\sigma}$$

$$d_n = \lim_{Q^2\to 0}\frac{F_3(Q^2)}{2m_N}$$

Need $Q^2 \to 0$ extrapolation

Problem: Prior to 2017, a spurious mixing between EDM and magnetic moments in all previous lattice computations of nucleon form factor.

Spurious mixing problem in lattice form factor method [M. Abramczyk, et al, 2017]

- CP violating interaction induces a chiral phase :

$$\langle 0|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

$\tilde{u}_p$ is a solution spinor of the free Dirac equation: $(\not{p} - m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$

α is mixing angle (CP-violating mass correction)

- This mixing angle α has to be calculated, and rotated away to get "net" CP-violation effect.

$$\bar{\tilde{u}}_{p',\sigma'}\left[\tilde{F}_1\gamma^\mu + (\tilde{F}_2 + i\tilde{F}_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]\tilde{u}_{p,\sigma} \equiv \bar{u}_{p',\sigma'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u_{p,\sigma}$$

[Previous "lattice" parametrization prior to 2017]

$$(F_2 + iF_3\gamma_5) = e^{2i\alpha\gamma_5}(\tilde{F}_2 + i\tilde{F}_3\gamma_5) \quad \Rightarrow \quad \begin{aligned}[F_2]_{\text{correct}} &= \tilde{F}_2 + \mathcal{O}(\alpha^2)\\ [F_3]_{\text{correct}} &= \tilde{F}_3 + 2\alpha F_2\end{aligned}$$

Previous lattice EDM results (prior to 2017) were subject to large contamination from F2,3 mixing.

Reanalysis of "lattice" θ induced EDM

[M. Abramczyk, et al, 2017]

Correction is simple: $[F_3]_{\mathrm{correct}} = \tilde{F}_3 + 2\alpha F_2$

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−1.50(16)	−0.217(18)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.560(40)	−0.247(17)	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−1.491(22)	−0.079(27)	−0.375(48)	−0.130(76)
	n	360	1.138(13)	−1.473(37)	−0.092(14)	−0.248(29)	0.020(58)

Removing spurious contributions : no signal of EDM

→ consistent with phenomenological estimates

How to improve the signal?

Noise reduction technique for θ-induced EDM

Noise reduction technique for θ-EDM

■ Constrain Q sum to fiducial volume for θ-EDM :

Topological charge: $Q \sim \int_{V_4} G\tilde{G}, \quad \langle Q^2 \rangle \sim V_4$

$Q \sim \int_{V_Q} d^4x q(x)$ (Statistical error^2 $\sim$ V_4)

- **in time around current** $|t_Q - t_J| < \Delta t$ [E. Shintani et al (2015), B. Yoon et al (2019)]
- **4d spherical around sink** $|x_Q - x_{sink}| < R$ [K. -F. Liu et al (2017)]
- **4d “cylinder”** $V_Q : |\vec{x}| < r_Q, \quad -\Delta t_Q < t_0 < T + \Delta t_Q$ [S. Syritsyn et al (2018)]
- **in time around source** $|t_Q - t_{\rm src}| < \Delta t$ [J. Dragos et al (2019)]

Selected recent progresses for θ-EDM will be shown.

1: α-improvement

[J. Dragos et al, arXiv:1902.03254]

- **3-pt functions with topological charge density**

$$\Delta C_{3pt}(\tau) \equiv \langle T\{N(T)\bar{Q}(\tau)\bar{N}(0)\}\rangle, \qquad \bar{Q}(\tau) \equiv \int d^3x G\tilde{G}(x,\tau)$$

- **Performing the spectral decomposition**

(1) $0 < \tau < T$

$$\Delta C_{3pt}(\tau) = \langle N(T)\bar{Q}(\tau)\bar{N}(0)\rangle \sim \sum_{n,m} e^{-E_n(T-\tau)-E_m\tau}\langle 0|N|n\rangle\langle n|\bar{Q}|m\rangle\langle m|\bar{N}|0\rangle \sim \sum_{m\neq n}\cosh\left(\Delta m_{mn}(\tau - T/2)\right)$$

$\langle N_+|\bar{Q}|N_+\rangle = 0$ due to P sym.

$(|N_+\rangle$: ground state nucleon $)$

(2) $T < \tau$

$$\Delta C_{3pt}(\tau) = \langle \bar{Q}(\tau)N(T)\bar{N}(0)\rangle \sim \sum_{n,m} e^{-E_n\tau - E_m T}\langle 0|\bar{Q}|n\rangle\langle n|N|m\rangle\langle m|\bar{N}|0\rangle \sim \sum_n e^{-E_n\tau}, \quad (E_0 \sim m_{\eta'})$$

exponentially suppressed

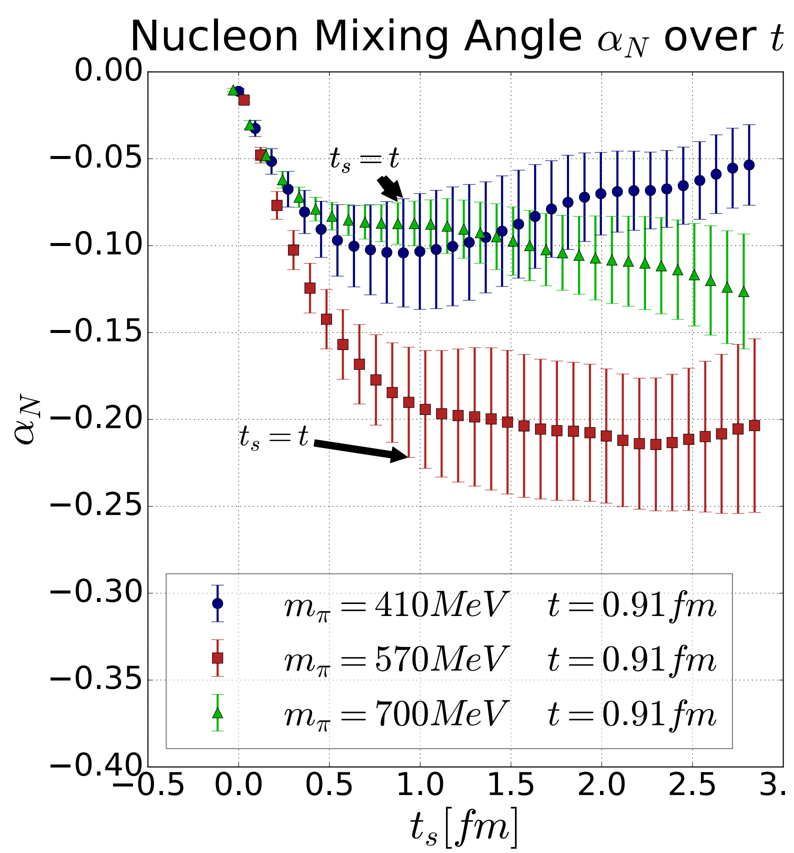
- **mixing angle α is obtained by fit analysis**

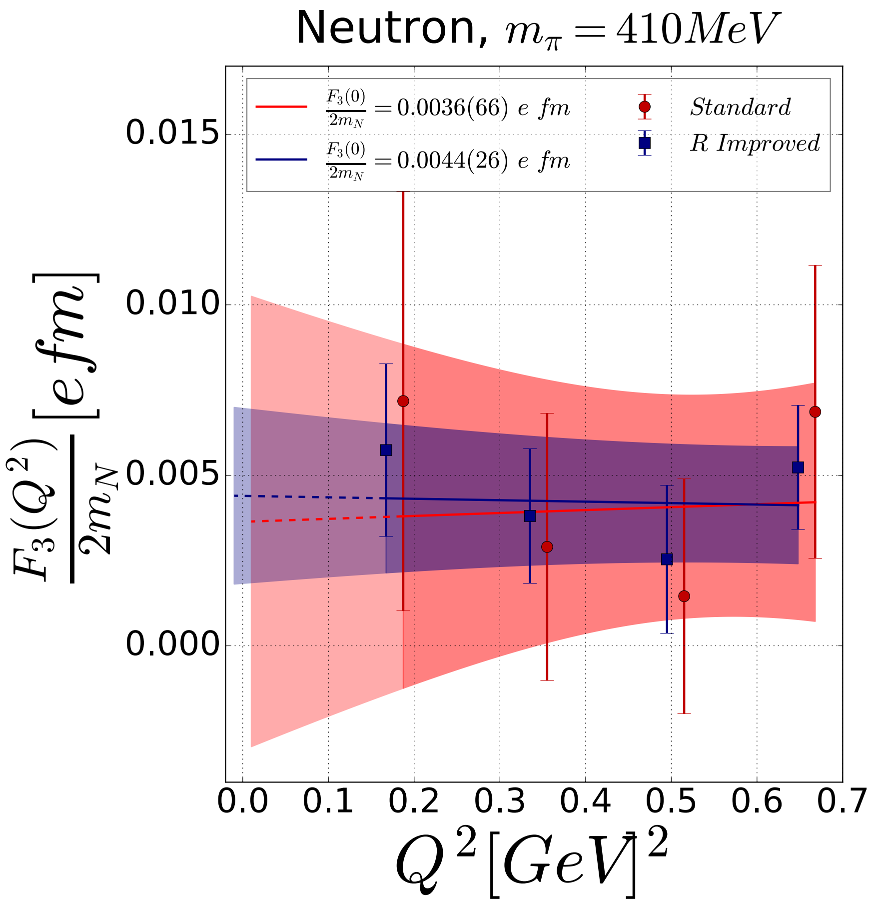
$$C_{3pt}(t_s) = \sum_{\tau=-t_s}^{\tau=t_s} \Delta C_{3pt}(\tau) = A + Be^{-Et_s} \quad (t_s >> T)$$

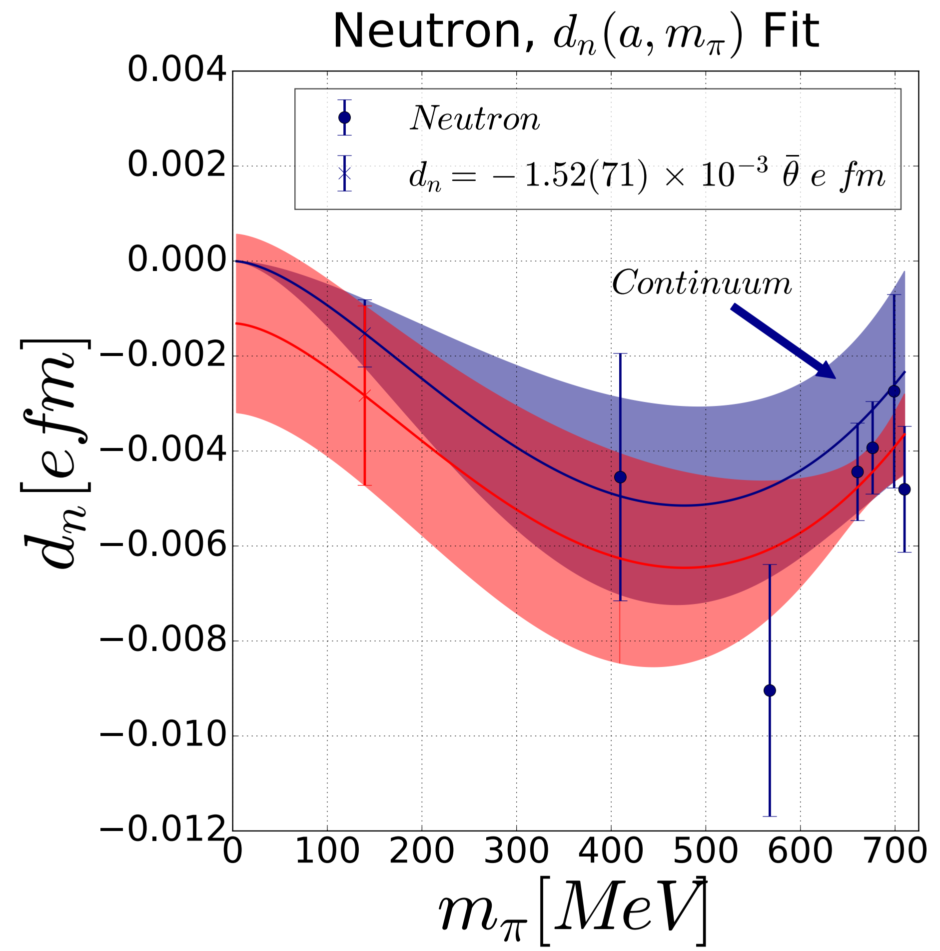
1: α-improvement and ChPT fit for F_3

[J. Dragos et al, arXiv:1902.03254]

$$t = T = t_{\rm sink} - t_{\rm src}$$







Fit ansatz: $$d_{p/n}(a, m_\pi) = C_1\ m_\pi^2 + C_2\ m_\pi^2 \log\left(\frac{m_\pi^2}{m_{N,phys}^2}\right) + C_3 a^2$$

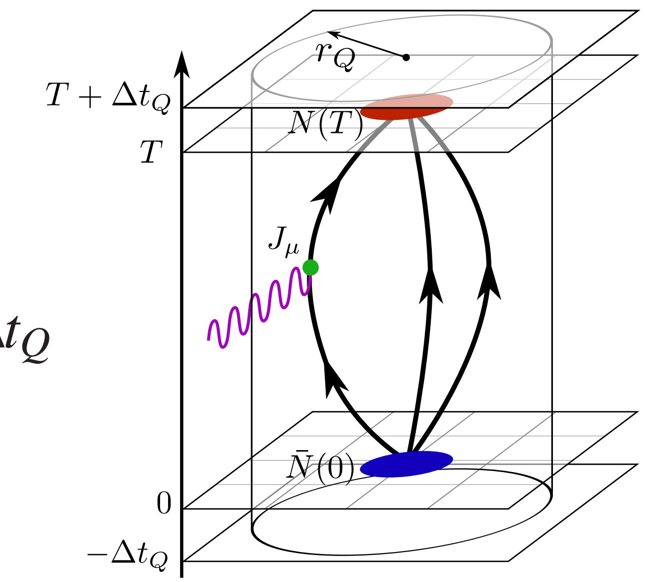
Non-zero signal at physical point for θ-EDM by extrapolation.

Far from chiral regime?

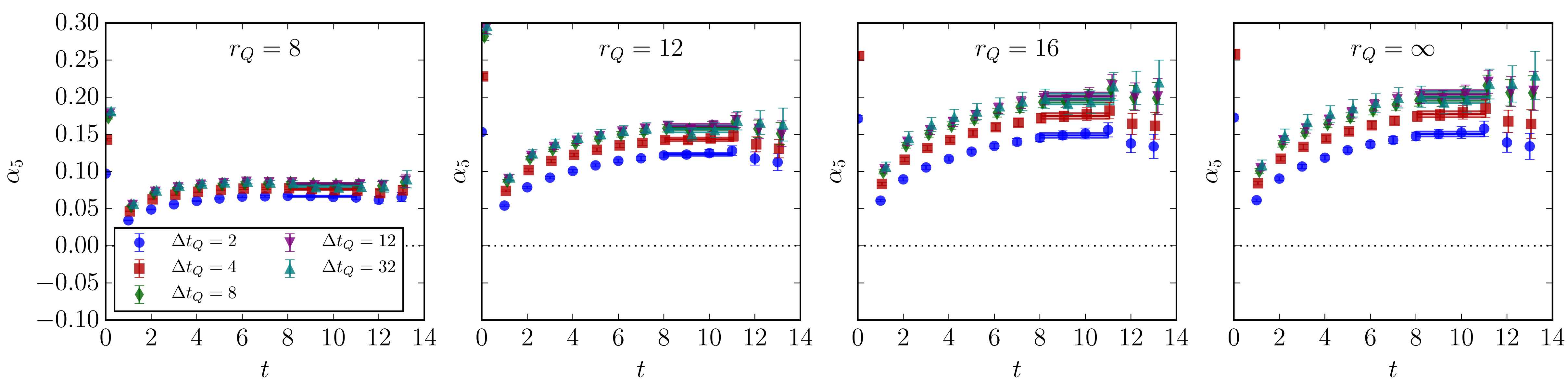
2. Our work

- Nf=2+1 (Mobius) Domain wall fermion, Iwasaki gauge action
- Reduced topological charge density $\tilde{Q}(\Delta t_Q, r_Q)$

$$\tilde{Q}(\Delta t_Q, r_Q) = \frac{1}{16\pi^2} \sum_{x \in V_Q} \mathrm{Tr}\left[\hat{G}_{\mu\nu}\tilde{\hat{G}}_{\mu\nu}\right]_x, \quad (\vec{x}, t) \in V_Q : \begin{cases} |\vec{x} - \vec{x}_0| \leq r_Q, \\ t_0 - \Delta t_Q < t < t_0 + t_{\mathrm{sep}} + \Delta t_Q \end{cases}$$



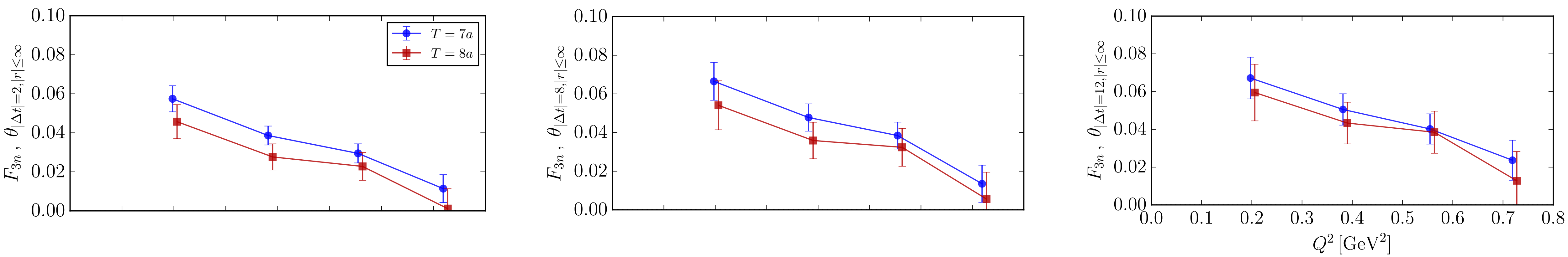
Convergence test of the parity-mixing angle from the reduced topological charge $\qquad m_\pi = 340$ MeV



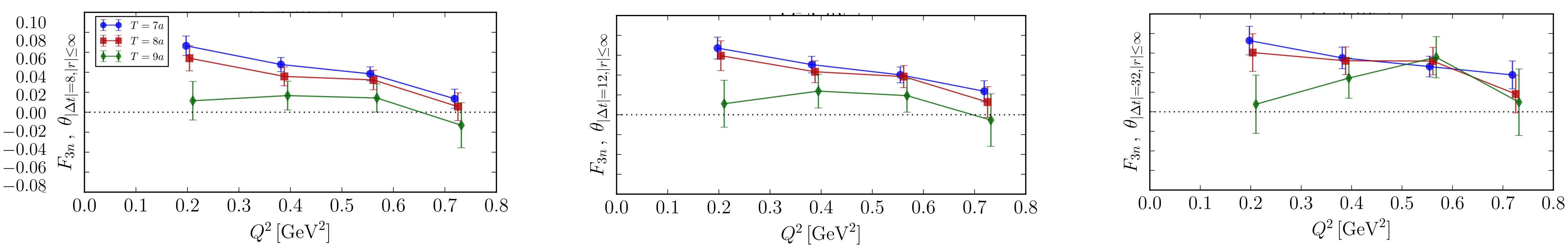
convergence : $r_Q \geq 16a,\ \Delta t_Q \geq 8a$

F_{3n} form factor

$m_\pi = 410$ MeV **1500 configs x 64 (AMA) samples = 96000 stat.**

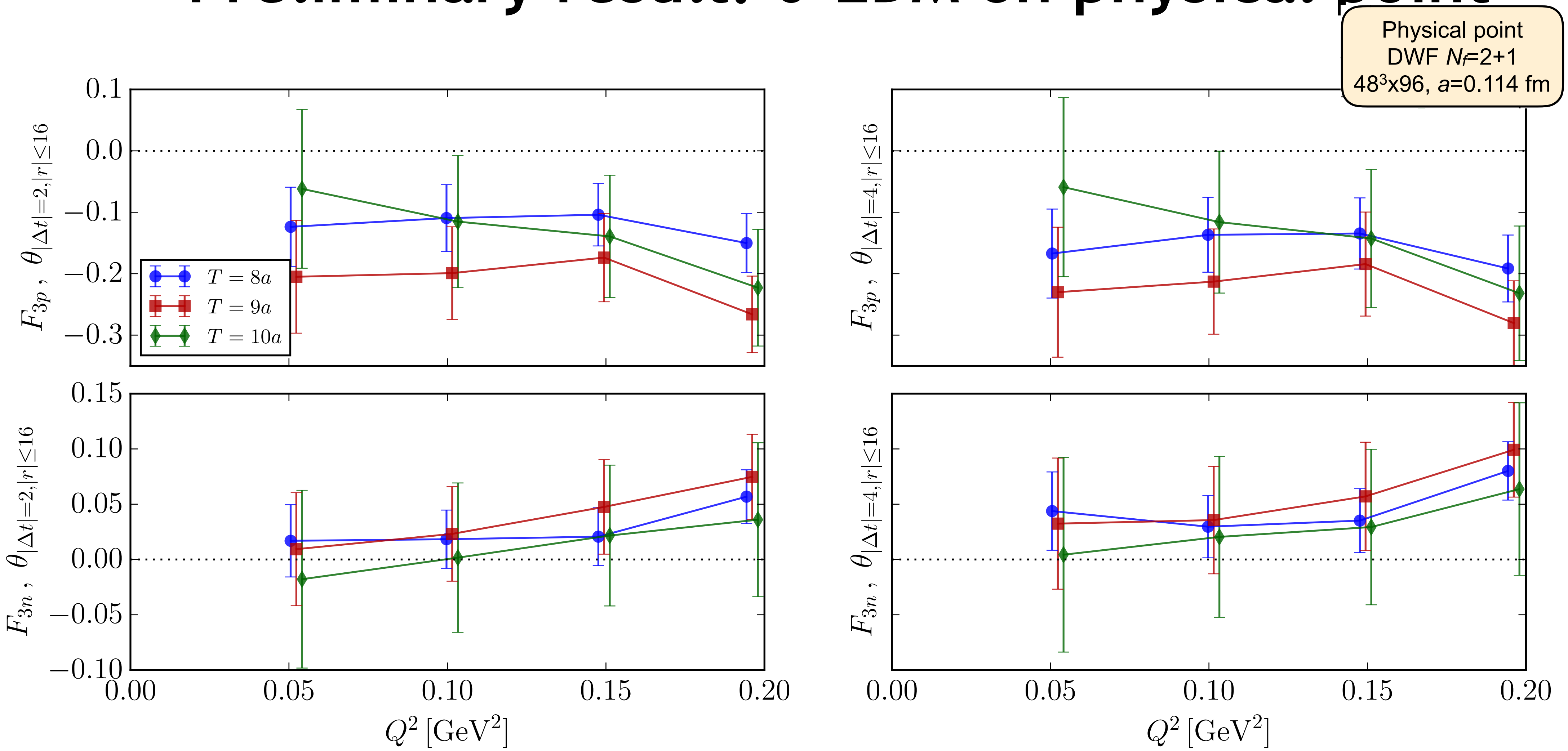


$m_\pi = 340$ MeV **1400 configs x 64 (AMA) samples = 89600 stat.**



convergence: $\Delta t_Q \geq 8a$ $\lim_{Q^2 \to 0} F_{3n}(Q^2) \sim 0.1$

Preliminary result: θ-EDM on physical point



33000 stat.

$F_{3n}(Q^2)$ **: consistent with zero.**

Our naive estimate of θ-nEDM at the physical point

- **Chiral fermion, m_π = 330 MeV (our result) :** $2m_N|d_n| = F_{3n}(0) \simeq 0.1$
- **scaling based on leading order ChPT:** $d_n \sim m_q \sim m_\pi^2$

$\Rightarrow$ $F_{3n}(0) \sim 0.02 \cdot \theta, \quad |d_n| \sim 0.002\ e\ \mathrm{fm} \cdot \theta$ **(physical point)**

- **Consistent with the results from QCD sum rule and the lattice result with ChPT fits.**

$d_n = -0.00152(71) e\ \mathrm{fm} \cdot \theta$ **[J. Dragos et al, arXiv:1902.03254]**

- **To constrain $|F_{3n}| < 0.02$ at m_{phys}, we need 25 ~ 100 times statistics ($\delta F_{3n}/F_{3n} \sim 5$ at physical point)**

Short summary : lattice θ-EDM calculations

- Various noise reduction techniques have been used, which in fact improve the signal-to-noise ratio in the form factor calculations.
- Clear signal at heavier mass with non-zero Q^2
- Result at the physical point has 50-100% error.

$$|d_n| = \mathcal{O}(10^{-3})\, e\, \mathrm{fm} \cdot \theta, \qquad |F_{3n}| = \mathcal{O}(10^{-2})$$

- There may be a tension between chiral (and Q^2) extrapolated value and a direct result at physical point.
- Need to understand π mass and Q^2 dependence of d_n.
- Constrain θ-induced nEDM at physical point is still challenging.

A new method

Matrix element approach with background electric field

Lattice QCD with background constant electric field

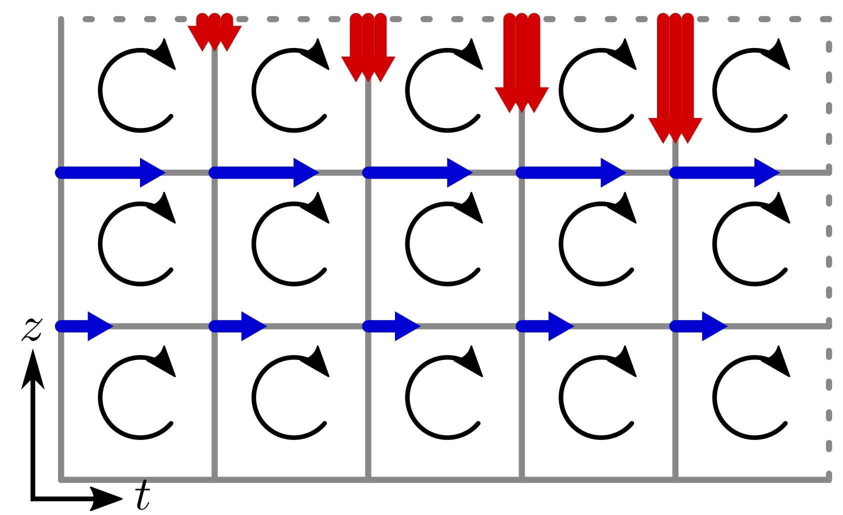
- Uniform electric field preserving translational invariance and periodic boundary conditions on a lattice (Euclidean imaginary electric field)
- used for the nucleon polarizability [W. Detmold, Tiburzi, and Walker-Loud, (2009)]
- No sign problem: Analytic continuation of CP-odd interaction
- consistency check of energy shift method and form factor method via cEDM operator.

$$U_\mu \to e^{iQ_q A_\mu} U_\mu$$

$$A_t(z,t) = \mathcal{E}_n z$$

$$A_z(z,t) = -\mathcal{E}_n L_z t \delta_{z=L_z-1}$$

Charge quantization due to finite volume.



strength of E field $\mathcal{E}_n = n\dfrac{6\pi}{L_z L_t}, \quad (n = \pm 1, \pm 2, \cdots)$

charge quanta $Q_q \mathcal{E}_n L_z L_t = 2\pi m, \quad (m : \text{integer})$

$(Q_u = 2/3, \quad Q_d = -1/3)$

24^3x 64 lattice minimal value of E (|n|=1)

$$\mathcal{E}_0 = \frac{6\pi}{L_z L_t} \sim 0.037 \text{ GeV}^2$$

$$\sim 186 \text{ MV/fm}$$

3pt function with topological charge density in the presence of background electric field

- **Consider 3-pt functions of topological charge density**

$$\Delta C_{3pt}(\tau,\vec{\mathcal{E}}) = \langle \hat{N}(T)\bar{Q}(\tau)\bar{\hat{N}}(0)\rangle_{\vec{\mathcal{E}}}, \quad (0<\tau<T)$$

- **Performing the spectral decomposition**

$$\Delta C_{3pt}(\tau,\vec{\mathcal{E}}) = \langle \hat{N}(T)\bar{Q}(\tau)\bar{\hat{N}}(0)\rangle_{\vec{\mathcal{E}}} \sim \sum_{n,m} e^{-E_n(T-\tau)-E_m\tau}\langle 0|\hat{N}|n,\mathcal{E}\rangle\langle n,\mathcal{E}|\bar{Q}|m,\mathcal{E}\rangle\langle m,\mathcal{E}|\bar{\hat{N}}|0\rangle$$

$$= |Z_N|^2 e^{-m_N T}\boxed{\langle N_+,\mathcal{E}|\bar{Q}|N_+,\mathcal{E}\rangle} + \text{(excited states)}$$

$|N_+,\mathcal{E}\rangle$ **: ground state nucleon in the presence of b.g. electric field**

This matrix element can be non-zero due to non-zero electric field, which corresponds to the energy shift (δE)

$$\boxed{\langle N_+,\mathcal{E}|\bar{Q}|N_+,\mathcal{E}\rangle = \delta E = d_n \times \vec{\Sigma}\cdot\vec{\mathcal{E}}}$$

c.f. 1st order energy correction in the perturbation theory of quantum mechanics

$$\hat{H} = \hat{H}_0 + \delta\hat{H}, \qquad \delta E_n = \langle n|(\delta\hat{H})|n\rangle$$

Numerical study

Numerical test of Matrix element method

- Nf=2+1 (Mobius) Domain wall fermion, Iwasaki gauge action
- Topological charge : operator improvement via gradient flow
- 2 gauge ensembles with two pion masses

  Mπ=330 MeV : ~1400 configs x 64 AMA samples -> 89.6k stat.

  Mπ=430 MeV : ~800 configs x 64 AMA -> 51.2 k stat.
- Background electric field : z-direction,
- Electric charge quanta: $|\mathcal{E}| = \pm 1, \pm 2$ in units of $\mathcal{E}_0 = \frac{6\pi}{L_z L_t}$
- Ratio method (3pt/2pt)

$$\frac{\mathrm{Tr}[T_{Sz\pm}\Delta C_{3pt}(T,\tau,\mathcal{E}_z)]}{\mathrm{Tr}[T_p C_{2pt}(T,\mathcal{E}_z)]} = \pm\delta E \quad (T \to \infty)$$

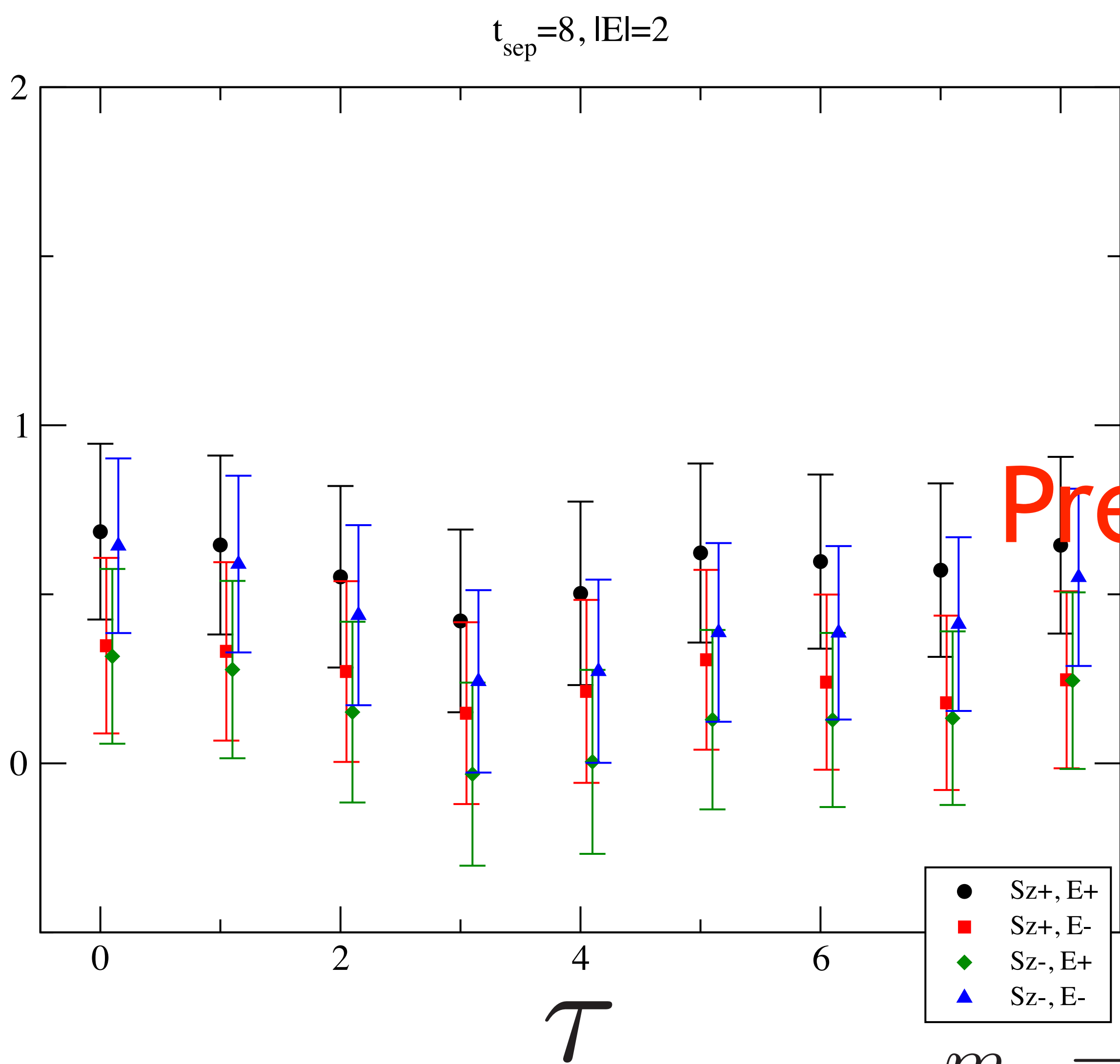
$$\Delta C_{3pt}(T,\tau,\vec{\mathcal{E}}) = \langle \hat{N}(T)\bar{Q}(\tau)\bar{\hat{N}}(0)\rangle_{\vec{\mathcal{E}}}, \qquad T_{Sz\pm} = \frac{1+\gamma_4}{2}(1\pm\Sigma_z), \qquad T_p = \frac{1+\gamma_4}{2}$$

$$\delta E = \frac{d_n \mathcal{E}_z}{2m_N} = \frac{F_{3n}(0)\mathcal{E}_z}{2m_N} \quad \Rightarrow \quad |F_{3n}(0)| = \frac{2m_N \delta E}{|\vec{\mathcal{E}}|}$$

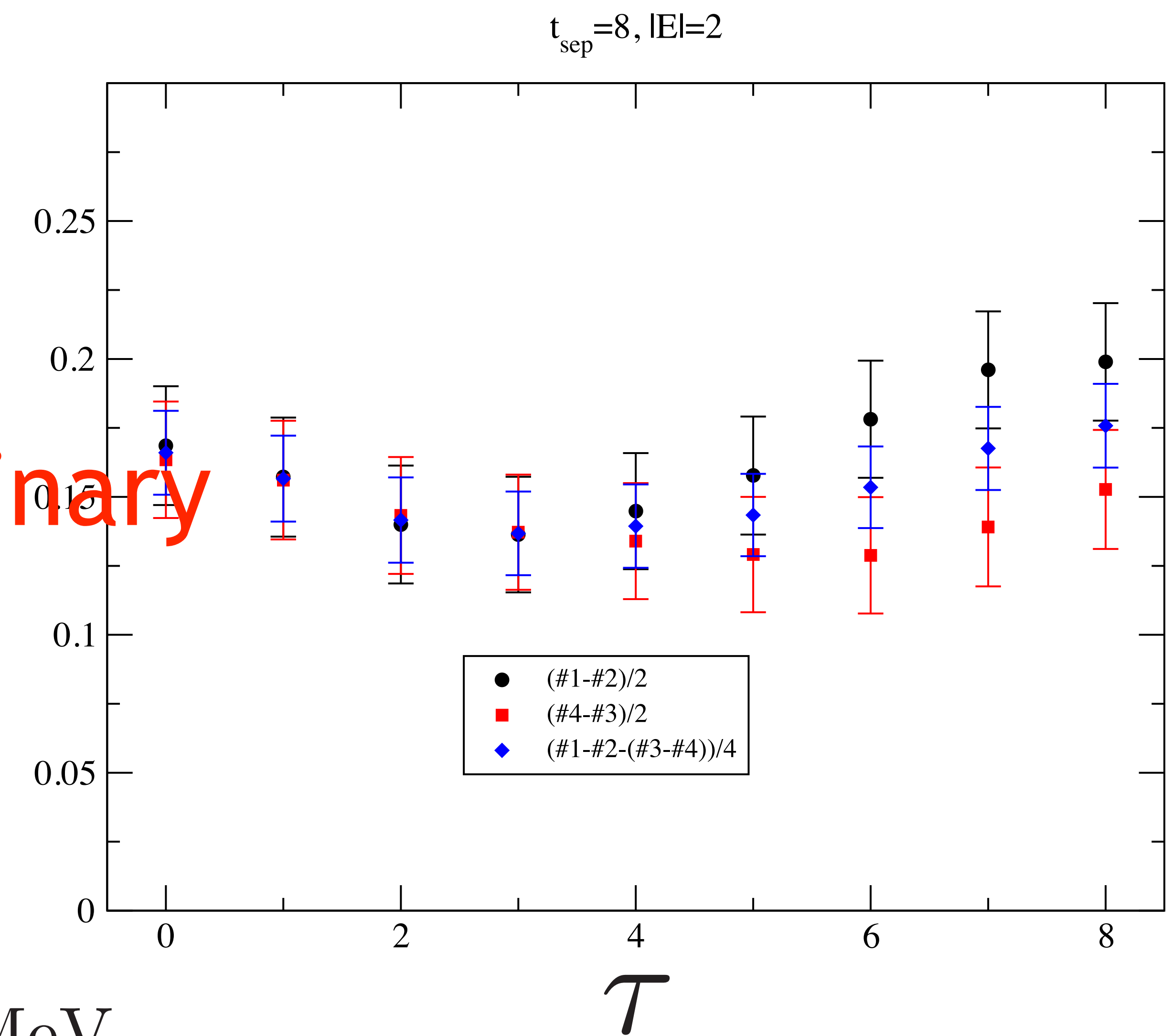
Ratio at mπ=340 MeV

$$|F_{3n}(0)| = \frac{2m_N \delta E}{|\vec{\mathcal{E}}|}$$

$$T = 8$$
$$|\mathcal{E}| = 2$$



Preliminary



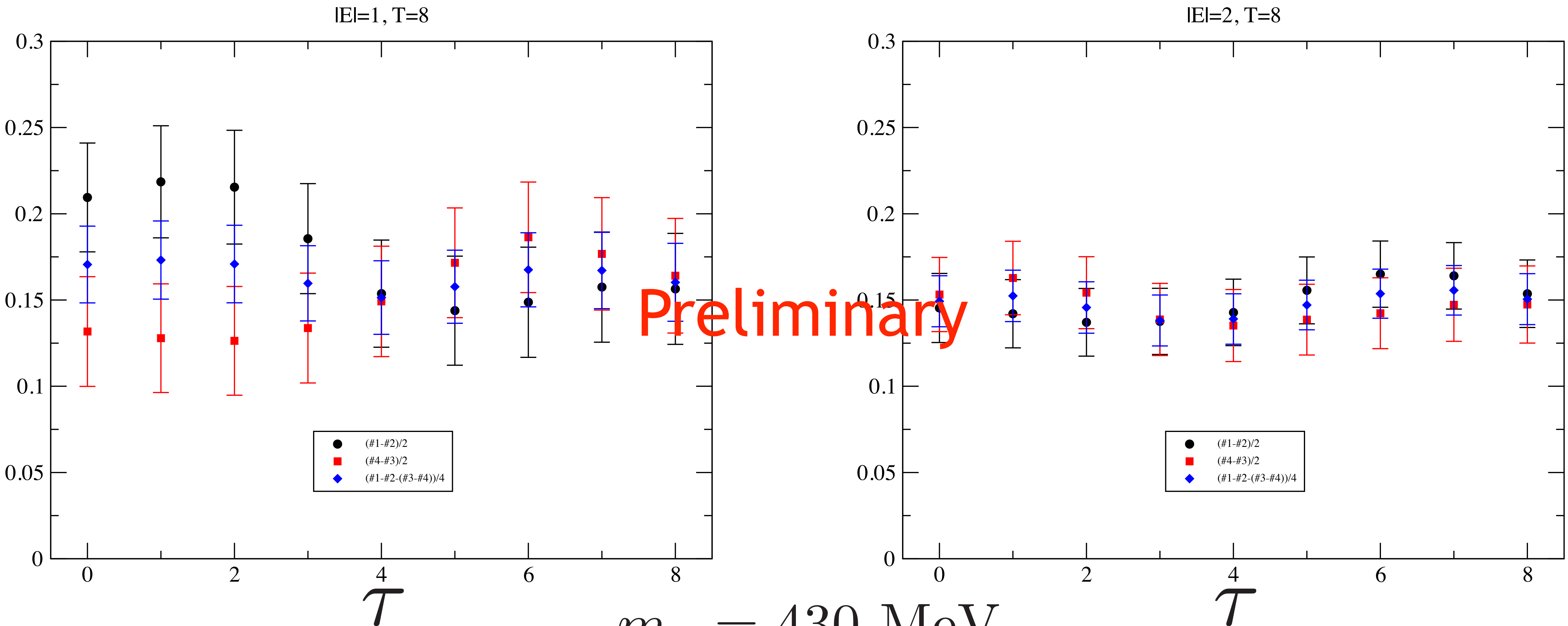
$$m_\pi = 340 \text{ MeV}$$

Difference between spin up (positive E) and spin down (negative E) components has better signal.

Good plateau.

Electric field dependence at mπ=430 MeV

$|\mathcal{E}| = 1$ $\qquad\qquad$ $|\mathcal{E}| = 2$



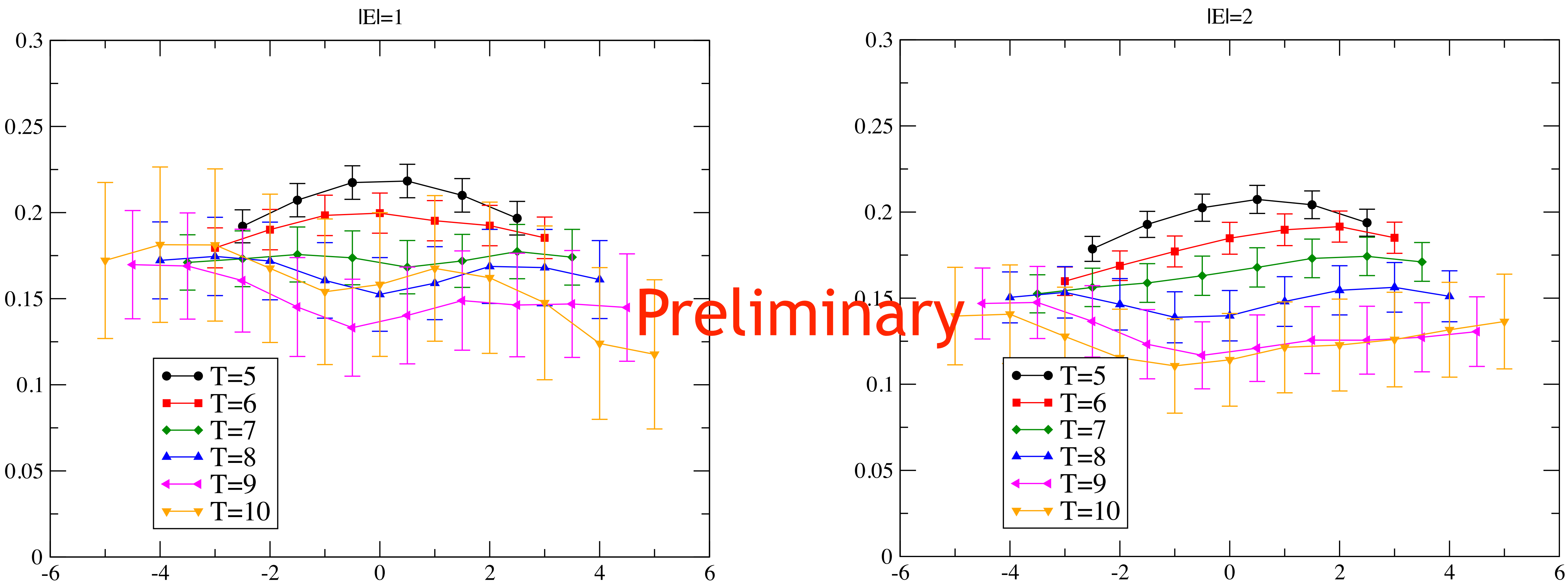
$m_\pi = 430\ \mathrm{MeV}$

Two results for |E|=1 and |E|=2 are consistent.

High order corrections (E^2) are small.

T dependence at mπ=430 MeV $\langle \hat{N}(T)\bar{Q}(\tau)\bar{\hat{N}}(0)\rangle_{\vec{\mathcal{E}}}$

$|\mathcal{E}| = 1$ $|\mathcal{E}| = 2$



$\tau - T/2$ $m_\pi = 430$ MeV $\tau - T/2$

Consistent results are obtained for T ≧ 8.

We obtain $F_{3n}(0) = \begin{cases} 0.15(2) & (|\mathcal{E}| = 1) \\ 0.14(1) & (|\mathcal{E}| = 2) \end{cases}$ at mπ=430 MeV

Summary

Lattice computation of lattice θ-EDM is very challenging.

Form factor methods

- Noise reduction techniques for Q-samplings have been developed in recent years.
- Good signal at heavier pion mass region
- The error becomes larger at the physical point.
- Need to understand π mass (and Q^2) dependence of $F_3(Q^2)$ form factor

A new method -matrix element approach-

- Potentially better control of the uncertainties (no need Q^2 extrapolation, no need to extend outside sink-source position)
- Need to study at the physical point

Thank you